appropriate formal solution of (2) is [3]

$$e_z = A\left(C_{e0}(\xi,q) - \frac{C_{e0}(\xi_0,q)}{M_{e0}^{(2)}(\xi_0,q)}M_{e0}^{(2)}(\xi,q)\right)c_{e0}(\tau,q) \quad (4)$$

where $C_{e0}(\xi,q)$ is modified Mathieu functions of the first kind, $c_{e0}(\tau,q)$ is a periodic Mathieu function of the first kind, and $M_{e0}^{(2)}(\xi,q)$ is a second solution of a Mathieu equation. The function $M_{e0}^{(2)}(\xi,q)$ corresponds to the Hankel function $H_0^{(2)}(z)$, where the $H_0^{(2)}(z)$ is used to represent outgoing waves in problems pertaining to a circular cylinder. From Maxwell's curl equation we have

$$h_\tau = \frac{1}{\iota\omega\mu_0(\mu^2-\kappa^2)l}\left(-\iota\kappa\frac{\partial e_z}{\partial\tau} + \mu\frac{\partial e_z}{\partial\xi}\right) \quad (5)$$

where $l = h/\sqrt{2}(\cosh(2\xi)-\cos(2\tau))^{1/2}$. The magnetic field h_τ at the surface of the elliptic cylinder is equal to the surface currents j_s. Then, for the total current j, we have

$$j = \oint j_s l\,d\tau = 2\int_0^\pi h\tau(\tau)l\,d\tau. \quad (6)$$

From (6) we have a relation between j and the coefficient A of (4). The radiation resistance R_m is defined by the ratio the radiative power to the square of the current flowing into the metal cylinder. The radiative power flow is given by

$$P_\xi^{sc} = \mathrm{real}\left(-e_z^{sc}\widetilde{h_\tau^{sc}}\right) \quad (7)$$

where, $\widetilde{h_\tau^{sc}}$ is the complex conjugate of the h_τ^{sc}, and

$$e_z^{sc} = -A\frac{C_{e0}(\xi_0,q)}{M_{e0}^{(2)}(\xi_0,q)}M_{e0}^{(2)}(\xi,q)c_{e0}(\tau,q) \quad (8)$$

and $\widetilde{h_\tau^{sc}}$ can be derived from (5) by making use of (8). Thus, the radiation resistance is defined by

$$R_m = \frac{2\int_0^\pi p_\xi^{sc} l\,d\tau}{|j|^2}. \quad (9)$$

From (7), (8), and (9), we have

$$R_m = -\frac{\left(\omega\mu_0(\mu^2-\kappa^2)\right)^2}{(2\pi\mu)^2} \cdot \frac{1}{\left|\frac{\partial C_{e0}(\xi_0,q)}{\partial\xi} - \frac{C_{e0}(\xi_0,q)}{M_{e0}^{(2)}(\xi_0,q)}\frac{\partial M_{e0}^{(2)}(\xi_0,q)}{\partial\xi}\right|^2} \cdot \frac{1}{\iota\omega_{\mu 0}\mu_\perp}\cdot\left|\frac{C_{e0}(\xi_0,q)}{M_{e0}^{(2)}(\xi_0,q)}\right|^2 \cdot M_{e0}^{(2)}(\xi,q) \cdot \widetilde{\frac{\partial M_{e0}^{(2)}(\xi,q)}{\partial\xi}} \cdot I \quad (10)$$

where

$$I = 2\int_0^\pi |c_{e0}(\tau,q)|^2\,d\tau. \quad (11)$$

It is to be noted that there is no ξ dependence of R_m in the ξ direction.

In Fig. 2, the radiation resistance R_m is plotted as a function of frequency. In Fig. 2, discontinuities of the curves near the frequency of 2.5 GHz are originated from the use of the approximate formulas for Mathieu functions. For small values of q, we expect the line of the curves in the low frequency regions to be more accurate, while the curves in the high frequency regions should be more reliable for large values of q. Interesting characteristics are shown in Fig. 2 that this transducer near the critical frequency $\omega = \gamma(BH)^{1/2}$. The following trends can be found for the range of parameters considered in this paper: 1) the exciting bandwidth of low frequency side decreases approximately proportional to x_0; 2) the maximum value of R_m decreases with increasing x_0; and 3) a number of zeros of the R_m appear as x_0 increases.

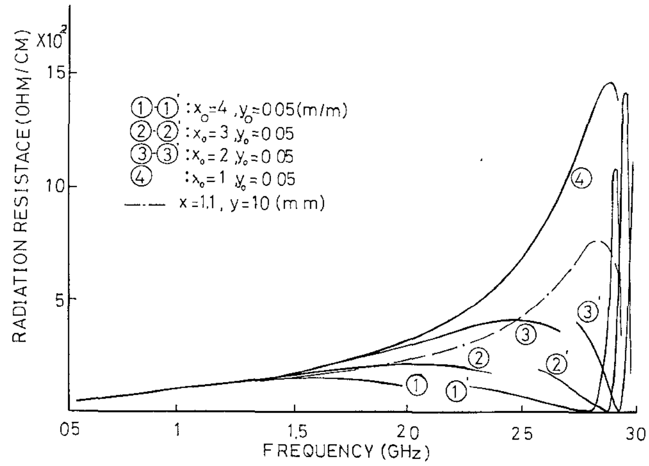

Fig. 2. Radial resistance is plotted as a function of frequency for various dimensions of the thin wire. Other parameters are biasing dc magnetic field $H = 500$ oe, saturation magnetization $4\pi M_s = 1800$ G.

In order to confirm this theory, a curve is plotted in Fig. 2 by a chain line so that this curve shows the resistance of a thin wire with a shape which is almost considered as a circular cylinder ($x_0 = 1.1$ m/m, $y_0 = 1.0$ m/m). This curve agrees quite well with the previous results which were obtained by using Bessel functions. [1]

III. Conclusion

In conclusion, a broad bandwidth (1 ~ 2 GHz) radial-wave transducer can be expected to design by making use of a fine wire with elliptical cross section. In view of the above, developments of a suitable radial line should produce extremely wide bandwidth and a magnetically tunable low frequency (0.5 ~ 1.0 GHz) microwave transducer.

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A Simple Formula for the Capacitance of a Disc on Dielectric on a Plane

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Abstract —**There is presented a simple explicit formula for the capacitance of a thin circular disc on a dielectric substrate on a plane (so-called "microstrip"). It gives continuous coverage of all shape ratios (r/h) and all**

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dielectrics (k). The formula is developed by transition between asymptotic formulas for small and large discs, and by interpolation between bounding formulas for lo-k and hi-k. The presentation and development are facilitated by a reference formula which is the sum of the C of the disc radius and that of the disc area (for any k). The relative error of C is less than 0.004 for all conditions.

Indexing terms—**Capacitance, disc, dielectric, electric-field theory, microstrip, printed circuits, curve fitting, mathematical techniques.**

I. Introduction

The capacitance of a thin circular disc to a parallel plane has been difficult to evaluate. It is not susceptible to the simple representation available for a two-dimensional model in terms of conformal mapping. Classic formulas have been stated for the extreme cases of a small disc or a large disc, relative to their separation. An interposed dielectric sheet (as in printed-circuit or "microstrip" technique) adds complication. During the past decade, laborious programs of numerical analysis have evaluated the intermediate shapes, even with the mixed dielectric.

There is presented here, a simple explicit formula for all shapes with interposed dielectric. A byproduct is a formula for a disc between parallel planes, with homogeneous dielectric. Each of these formulas is essentially an interpolation between the extremes of shape and of dielectric. The free-space formulas are within 0.003 of capacitance and the mixed-dielectric formulas are within 0.004.

The background is represented by references in several categories:

- [1] [2] the classical formulas for a small or large disc with free-space dielectric;
- [3] [6] related two-dimensional models of a strip near a plane, including mixed dielectric;
- [3] [7] a strip between planes, for the edge effect around a disc with hi-k dielectric;
- [4] [8] [9] numerical evaluation of intermediate shapes with mixed dielectric;
- [5] applications to a small antenna;
- [10] recent closer formula for a large disc.

Beyond the classics, the recent papers by Chew [8], [10] and Leong [9] have provided close support for the present formulation of intermediate shapes with dielectric. The pattern of this development is that of the author's formula for a strip line [3], [6].

Fig. 1 shows the essential dimensions of a thin circular disc:

- Near one parallel plane, with any dielectric interposed; or
- Between two parallel planes, with homogeneous dielectric (free space).

There will be evaluated in this order:

- C_1 = capacitance to one plane with free-space dielectric ($k=1$);
- C_2 = capacitance to two planes with homogeneous (free-space) dielectric;
- C_k = capacitance to one plane with interposed dielectric (k).

The second case is relevant in that it has the same field configuration as the limiting case of hi-k dielectric in the third case.

A feature of this development is the use of a reference value of C that is a fair approximation for all cases (within 0.1 of C) and asymptotically represents the extreme shapes. It is the sum of:

- The capacitance of a disc with large separation from the plane (s); and
- The capacitance of the disc area with small separation.

The close evaluation is achieved by a supplement to this reference value, needed most for intermediate shapes.

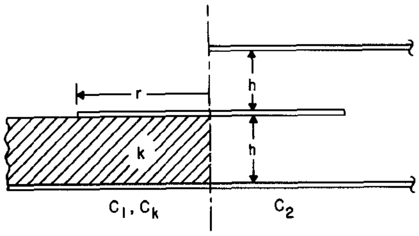

Fig. 1. Circular disc near one or two planes.

II. Symbols

Rationalized MKS units.

r	= radius of thin circular disc.
h	= height of disc from plane.
C	= capacitance of disc.
C_1	= C of disc to plane ($k=1$).
C_k	= C of disc to plane (any k).
C_2	= C of disc to two planes ($k=1$).
C_{1s}, C_{ks}, C_{2s}	= reference C for each case.
C_r	= C of disc radius (far from any plane).
C_a	= C of disc area (to plane).
$C(K)$	= C of large disc per Kirchhoff formula.
$C(KC)$	= C of large disc per Kirchhoff–Chew formula.
$C(C)$	= C of medium-to-large disc per Chew's table.
ϵ_0	= electricity (electric permittivity) of free space.
k	= ϵ/ϵ_0 = dielectric constant.
k_c	= modified k for interpolation.
m	= coefficient for potential at image location.
LB, UB	= lower and upper bounds.

III. A Reference for All Cases

A principal feature of this presentation is the choice of a reference for critical comparison of various formulas. This reference is the sum of the formulas for the extremes of small and large discs.

Referring to Fig. 1, with a small disc over one plane, the radius gives

$$C_r = \epsilon_0 \frac{1+k}{2} 8r. \tag{1}$$

With a large disc, the area gives

$$C_a = \epsilon_0 k \pi r^2 / h. \tag{2}$$

The sum of these two is taken as a reference

$$C_{ks} = C_r + C_a = \epsilon_0 r \left(\frac{1+k}{2} 8 + k\pi r/h \right) = C_{1s} \text{ if } k=1. \tag{3}$$

This matches the asymptotic values at both extremes and is within 0.10 of the correct value for all shapes and dielectric k. Incidentally, it is a useful approximation for estimates.

Referring again to Fig. 1, the disc between planes is to be only with homogeneous, anisotropic dielectric, so free-space dielectric is taken as representative. The reference sum becomes

$$C_{2s} = C_r + 2C_a = \epsilon_0 r (8 + 2\pi r/h). \tag{4}$$

This has similar attributes.

The relevant reference will be used for graphical presentation of each case. It enables an expanded picture of small variations.

IV. A Disc Near One Plane

First, the disc is separated from a parallel plane by free-space dielectric (ϵ_0 as in air). There are three cases:

- An asymptotic formula for a small disc ($r \ll h$);

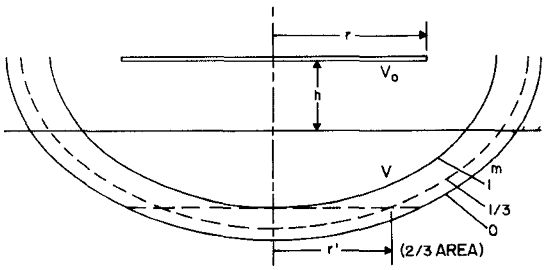

Fig. 2. Image in one plane.

- An asymptotic formula for a large disc ($r \gg h$);
- Computations for intermediate size.

The extreme cases are susceptible to analytic formulation. The intermediate case has been evaluated by various methods to give some discrete values. The latter will be used to devise a transition formula which is close for all shapes.

For a small disc, we start with its capacitance in free space, far from the plane, as determined by the radius

$$C_r = \epsilon_0 8r. \tag{5}$$

For the effect of the plane, we take an image in the plane and apply some simple rules.

Fig. 2 shows the disc and its free-space potential at the location of its image in the plane. The disc is charged to a potential V_0, giving contours of lesser potential V. One contour intersects the image location at a radius r'. The average potential at the image location is defined with weighting proportional to the charge density, taken to be the same as on an isolated disc. This average is found to be that of a contour including 2/3 of the area of the disc-image circle.

The use of a spheroidal contour and this average is a feature of this derivation. The voltage ratio at the radius r' is given by Smythe [2]:

$$V/V_0 = \frac{2}{\pi}\,\mathrm{atan}\frac{r/h}{\sqrt{\left[2-\frac{m}{2}(r/h)^2\right]+\sqrt{\left[2-\frac{m}{2}(r/h)^2\right]^2+4(r/h)^2}}} \tag{6}$$

in which $m = 1-(r'/r)^2$, so the weighted average requires $m = 1/3$.

It can be shown that the capacitance between disc and plane is expressed by this formula with some value of m ($0 < m < 1$):

$$C_1(m) = \epsilon_0 8r/(1 - V/V_0)$$

$$= \epsilon_0 8r \Big/ \frac{2}{\pi} \cdot \mathrm{atan}\left(\frac{h}{r}\sqrt{\left[2-\frac{m}{2}(r/h)^2\right]+\sqrt{\left[2-\frac{m}{2}(r/h)^2\right]^2+4(r/h)^2}}\right). \tag{7}$$

With $m = 1/3$, this will appear to give a very close approximation for the range, $0 < r/h < 2$.

Fig. 3 shows various formulas plotted in terms of difference from the described reference (3). It includes four graphs for a small disc:

- the isolated disc for extreme LB;
- $m = 0$ for closer LB;
- $m = 1/3$ for closest approximation;
- $m = 1$ for UB.

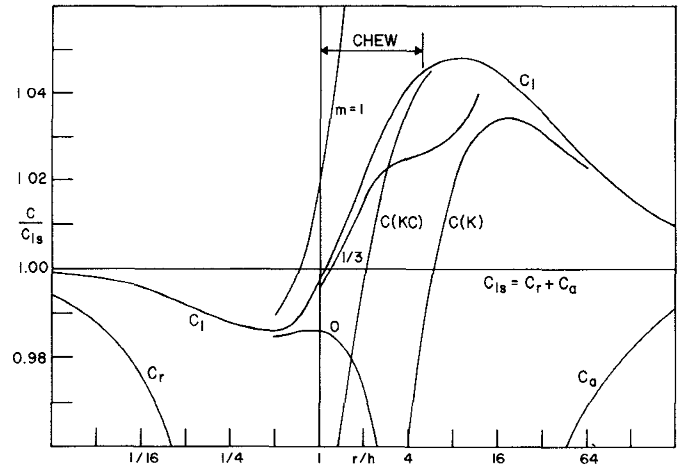

Fig. 3. Disc near one plane (C_1)

For a large disc, we start with the classic formula of Kirchhoff [1] for free-space dielectric:

$$C_1(K) = C_a[1 + (2h/\pi r)(\ln 8\pi r/h - 1)]$$
$$= \epsilon_0 r[\pi r/h + 2(\ln 8\pi r/h - 1)]. \tag{8}$$

The two parts are the area and the first-order edge effect. Recently Chew [10] has made a major contribution by adding another term in the asymptotic formula of Kirchhoff. The result is

$$C_1(KC) = C_1(K) + C_a(h/\pi r)^2(\ln^2 8\pi r - 2)$$
$$= C_1(K) + \epsilon_0 \frac{1}{\pi} h(\ln^2 8\pi r - 2). \tag{9}$$

This carries a comparable degree of approximation down to 1/3 the radius.

Fig. 3 includes three graphs for a large disc:

- the disc area for extreme LB;
- (K) for Kirchhoff's LB;
- (KC) for Chew's closer LB.

For a disc of intermediate size, we have no simple formula that is derived from basic principles. Recently Chew [8] has computed a close approximation by a numerical method based on "dual integral equation formalism". While directed to a large disc, he has provided a table of examples including intermediate size ($1 < r/h < 10$). It happens that this table covers the transition between the closest formulas for small disc and large disc, as indicated in Fig. 3.

From this information, the following interpolation formula has been devised to match the closest approximations in the three regions (with $k = 1$):

$$C_1 = \epsilon_0 r\left(\pi\frac{r}{h} + 8 + \frac{2}{3}\ln\frac{1 + 0.8(r/h)^2 + (0.31\,r/h)^4}{1 + 0.9(r/h)}\right). \tag{10}$$

This is plotted as the main graph in Fig. 3. The first and second parts are the reference value so the third part provides the deviation therefrom. The relative deviation being small (-0.015 to $+0.05$), the third part is not critical. Its form is chosen to retain several terms of the known asymptotic approximations at both extremes. Then its coefficients are subjected to one adjustment to match Chew's table for intermediate shapes while retaining the asymptotic approximations. The result is a remarkably close approximation for the entire range. It is within 0.003 of the known values.

V. A Disc Between Two Planes

In the author's previous treatment of a strip on a dielectric sheet on a plane, the limiting case of hi-k dielectric was noted to have the same field pattern as either side of a strip between two planes [3], [6], [7]. The same rule is applicable to the subject disc. Therefore, the disc between two planes, as shown in Fig. 1, is here formulated. The approach is generally similar to that for one plane. In Fig. 4, these formulas are plotted in terms of difference from the relevant reference (4). Here the reference provides an UB and the difference will appear to be small (<0.10).

For a small disc, in the limit, the two planes are no different from one (5). For the effect of two planes, we take a series of images of alternating sign. Each one is evaluated as in Fig. 2 and (6). Then their sum determines the C of the disc between the two planes, an extension of (7).

The result is plotted in Fig. 4, which will appear to provide a close LB in the transition region.

A simple formula for UB can be derived from the sum for the limiting case of a small disc

$$UBC_2 = C_r/[1-(2/\pi)(r/h)\ln 2]. \tag{11}$$

This is plotted as an UB in Fig. 4.

For a large disc, the classic approximation is the addition of $(h/\pi)\ln 4$ to the radius to get the effective radius. The resulting formula is

$$UBC_2 = 2C_a[1+(1/\pi)(\ln 4)(h/r)]^2. \tag{12}$$

This is plotted in Fig. 4 as an UB which also is close in the transition region.

At their closest approach, the relative separation of the closest bounds is only 0.015 so a smooth transition could leave an error much less than this separation.

The following formula has been devised to fit the asymptotic behavior at both extremes and to provide a smooth transition:

$$C_2 = C_{2s}[1-1/(4+2.6r/h+2.9h/r)]. \tag{13}$$

This is plotted as the main graph in Fig. 4. The second part is the difference below the reference value (4). It is symmetrical about $r/h=1.06$, the ratio mean between the asymptotic slopes. The residual error of this formula is estimated to be a small fraction of the minimum separation, say <0.002.

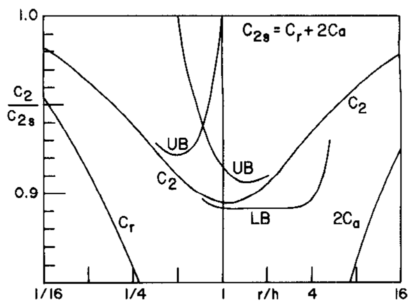

Fig. 4. Disc between two planes (C_2).

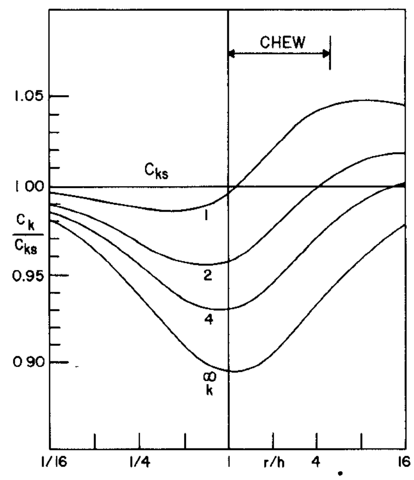

Fig. 5. Disc near one plane with dielectric (C_k).

VI. A Disc on Dielectric on a Plane

The choice of a reference value (3) gives a remarkable opportunity for presenting on one set of coordinates anything between the extremes of lo-k and hi-k. Fig. 5 shows these extremes as bounds of a rather small region that includes all values of r/h and k. A simple rule for interpolation will give a formula for all of this region.

In the earlier study of a strip line with similar dielectric [3], [6], a rule for interpolation was based on the concept of series-parallel C components contributed by the dielectric. The same concept is applied here.

For the extreme of hi-k, the capacitance becomes

$$C_k = k\left(\frac{1}{2}C_2\right). \tag{14}$$

This follows from the same shape of field pattern in the space between the disc plane and the lower plane, as seen in Fig. 1.

An interpolation between the bounds in Fig. 5 can be expressed in terms of a derived k_c which likewise varies between 1 and ∞ (see (3), (16), (10), (3), (13), and (4))

$$C_k/C_{ks} = (1/k_c)C_1/C_{1s} + (1-1/k_c)C_2/C_{2s}. \tag{15}$$

Chew's recent table [8] gives a close computation for some cases in this region which includes the most sensitive interpolation

$$r/h = 1,2,5,10; \quad k = 2.65, 9.6.$$

Closest match to these tables is obtained by making

$$k_c - 1 = 0.63(k-1); \quad k_c = 0.37+0.63k. \tag{16}$$

The factor 0.63 is conceptually an interpolation between series and parallel components of C in the dielectric.

Note that C_{1s} is (3) with $k=1$.

Formula (15) with (16) becomes an explicit formula for all cases on one plane. Its other ingredients are given by formulas (3) (4), (10), and (13). In Fig. 5 are graphed the bounds and two intermediate values of k in the region of most sensitive interpolation. A comparison with Chew's tables (in the indicated region which is most sensitive) and Leong's tables (over a wider region) gives agreement within 0.0034 of C. The residual error all over is taken to be within this discrepancy.

VII. Conclusion

A number of features, old and new, have been integrated to formulate the C of a disc over a plane with interposed dielectric. The principal sources are tabulated here.

disc:	small	medium	large
lo-k	new (7)	Chew [8]	Chew [10] (9)
med-k	Leong [9]	Chew [8]	Chew [8]
hi-k	new (14)		old (12)

The integration of these sources has been facilitated by the simple general reference (3) described as a basis for presentation. The result is a simple explicit formula (15) for continuous coverage of the entire range of shape and dielectric.

The initial impetus for this development came from a specific interest in a small antenna in the form of a disc-loaded monopole with no dielectric substrate [5]. From this, the author perceived the opportunity for the more general formula reported herein.

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Frequency Dependent Characteristics of Microstrips on Anisotropic Substrates

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***Abstract*—Frequency dependent characteristics are discussed for the microstrip line on anisotropic substrate (original line) from the extension of the results obtained in an isotropic substrate case. In approximating the original line by the equivalent line on an isotropic substrate, it is best to maintain h and w of the equivalent line equal to those of the original line. The reason is that the inflection frequency f_i is a function of w and h and that f_i plays an important role in calculating the dispersion. Three approximate dispersion formulas are derived owing to this idea. The results obtained by these formulas are compared with the other available theoretical and experimental results for sapphire substrates. Good agreement is seen.**

I. Introduction

Many articles have appeared giving design data for a microstrip which is an essential part of the integrated circuit in modern microwave devices. Sapphire has several advantages as a substrate material, so that its higher cost is offset [1]. Sapphire is a uniaxial crystal; it is therefore anisotropic. Being anisotropic, the microstrip lines possess characteristics which differs somewhat from that of lines on isotropic substrates.

The characteristics of microstrip lines on anisotropic substrates have been already investigated with a high degree of accuracy in the quasistatic approximation (see [9]–[11]). On the other hand, many available analyses for the case of isotropic substrate also can be used for the case of anisotropic substrate by using the transformation from anisotropic to isotropic problems [2], [4], [5], [10].

However, recent developments require the operation of a microstrip line at higher frequencies. Some authors have theoretically studied the frequency dependent characteristics for the case of anisotropic substrates [4], [5] and have checked the validity of the equivalent line of the isotropic substrate for such a case [4], [5]. However, there was a large discrepancy between the effective dielectric constants of the two corresponding lines at the higher frequency. The experimental investigation and the empirical formula have been compared to a sapphire substrate [3].

On the other hand, the computer-aided design of microstrip circuits requires accurate and reliable information on the dispersive behavior. For the case of an isotropic substrate, a few approximate formulas satisfying the CAD requirement have been derived (see [6], [12]). The author has also formulated a new approximate dispersion formula, and discussed the important role of the inflection frequency f_i on the dispersion curves, and the influence of the structural parameters, ϵ^*(relative dielectric constant), h(substrate thickness), and w/h(shape ratio) to the dispersion curves [12].

In this short paper, the simple approximate dispersion formulas are derived for a microstrip line on an anisotropic substrate. As an example, the numerical results of a case of sapphire substrate have been compared with the other available theoretical results [4], [5] and experimental data [3].

II. Approximate Dispersion Formula

Consider the microstrip line of width w on an anisotropic substrate of thickness h whose permittivity tensor (in two-dimensional space) is presented by

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_\xi^* & 0 \\ 0 & \epsilon_\eta^* \end{pmatrix} \epsilon_0 \tag{1}$$

where ϵ_ξ^* denotes the relative dielectric constant in the direction of the ξ-axis, ϵ_η^* denotes the relative dielectric constant in the direction of the η-axis, and ϵ_0 is the permittivity of free-space. The structure is shown in Fig. 1. The $\xi-\eta$ coordinates are identical to the principal axes of this substrate and can be obtained by rotating the $x-y$ coordinates with the angle γ. In designing such a line, the characteristic impedance Z and the phase velocity v(wave length λ, propagation constant β) must be obtained. Let us express the effective dielectric constant at a frequency f by $\epsilon_{\text{eff},f}^*$. These values above can be obtained as follows [7], [8], [12]:

$$Z = \frac{\eta_0 D}{\dfrac{C_0}{\epsilon_0}\sqrt{\epsilon_{\text{eff},f}^*}} \tag{2}$$

$$\frac{v}{v_0} = \frac{\lambda}{\lambda_0} = \frac{\beta_0}{\beta} = \frac{1}{\sqrt{\epsilon_{\text{eff},f}^*}} \tag{3}$$

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